Answer Key:

	A	B	C	D
1.	●	○	○	○
2.	○	●	○	○
3.	●	○	○	○
4.	○	○	○	●
5.	○	●	○	○
6.	○	●	○	○
7.	●	●	●	●
8.	●	●	○	○
9.	●	○	●	○
10.	●	●	●	○
11.	●	●	●	●
12.	○	○	○	●
13.	●	○	○	○
14.	○	○	○	●

15.	p	q	r	s	t	u
A	○	●	○	○	○	○
B	○	○	●	○	○	○
C	●	○	○	○	●	○
D	○	○	○	●	○	●

ANS	0	1	2	3	4	5	6	7	8	9
16.	○	○	●	○	○	○	○	○	○	○
17.	○	○	●	○	○	○	○	○	○	○
18.	○	○	○	○	●	○	○	○	○	○
19.	○	○	●	○	○	○	○	○	○	○

	A	B	C	D
20.	●	○	○	○
21.	○	●	○	○
22.	●	○	○	○

Detailed Solutions

1. Coordinates of a point on the line $y = x\sqrt{3}$ at a distance r from origin is $(r\cos\theta, r\sin\theta)$

$\because \tan\theta = \sqrt{3} \therefore \left(\frac{r}{2}, \frac{r\sqrt{2}}{2}\right)$ lies on the given curve

$$\Rightarrow \frac{r^3}{8} + \frac{r^3.3\sqrt{3}}{8} + 3.\frac{r}{3}.\frac{r\sqrt{3}}{2} + 5.\frac{r^2}{4} + 3.\frac{r^2.3}{4} + 4.\frac{r}{2} + 5.\frac{r\sqrt{3}}{2} - 1 = 0$$

$$\left(\frac{1+3\sqrt{3}}{8}\right)r^3 + \frac{r^2}{4}\left(3\sqrt{3}+14\right) = \frac{r}{2}\left(5\sqrt{3}+4\right) - 1 = 0$$

$$r_1.r_2.r_3 = \frac{8}{3\sqrt{3}+1} = \frac{8}{27-1} \times \left(3\sqrt{3}-1\right) = \frac{4}{13}\left(3\sqrt{3}-1\right)$$

2. $\int f^{-1}(x)\,dx = f^{-1}(x).x - \int x.(f^{-1}(x))'\,dx$

$= f^{-1}(x).x - \int f(t).dt$ put $f^{-1}(x) = t$

$\Rightarrow (f^{-1}(x))'\,dx = dt$

$= f^{-1}(x).x - g(t) = x.\,f^{-1}(x) = g(f^{-1}(x)) + c$

3. $S = \{H, TH, TTH, TTT\}$

$E \rightarrow$ head does not appear at first toss

$A \rightarrow$ coin is tossed thrice

$$P\left(\frac{A}{E}\right) = \frac{P(A \cap E)}{P(E)} = \frac{P(\{TTh, TTT\})}{P(\{TH, TTH, TTT\})}$$

4. $f'(x) = 6x + 3^2.4x^3 + ...$

$f'(x) = x(6 + 3^2.4x^2 + ...)$

$f'(x)$ changes sign at $x = 0$ only from negative to positive hence local minimum at $x = 0$

5. Let $\tan^{-1} x = \theta, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$3\theta + \cos^{-1}(\cos 3\theta) = 0$

$\cos^{-1}(\cos 3\theta) = -3\theta \Rightarrow -\pi \le 3\theta \le 0$

$\Rightarrow -\frac{\pi}{3} \le \theta \le 0 \Rightarrow x \in [-\sqrt{3}, 0]$. So number of integral solutions is 2.

6. Let $K \equiv (d, 0)$ any line through K is

$\frac{x-d}{\cos\theta} = \frac{y-d}{\sin\theta} = r$ i.e. $x = d + r\cos\theta,\ y = r\sin\theta$

It lies on the parabola

$\therefore r^2 \sin^2\theta = 4(d + r\cos\theta)$

i.e. $r^2\sin^2\theta - 4r\cos\theta - 4d = 0$

$\therefore KP + KQ = \frac{4\cos\theta}{\sin^2\theta}$ and $KP.KQ = -\frac{4d}{\sin^2\theta}$

$\therefore \dfrac{1}{PK^2}+\dfrac{1}{QK^2}=\dfrac{(KP+KQ)^2-2KP.KQ}{(KP.KQ)^2}$

$=\dfrac{\dfrac{16\cos^2\theta}{\sin^4\theta}+\dfrac{8d}{\sin^2\theta}}{\dfrac{16d^2}{\sin^4\theta}}=\dfrac{16\cos^2\theta+8d\sin^2\theta}{16d^2}$

$\dfrac{16+(8d-16)\sin^2\theta}{16d^2}$ is independent of $\theta \therefore d=2$

7. (A) $|adj(KA)|=|KA|^{n-1}=K^{n^2-n}.|A|^{n-1}$
correct option

(B) If $|A|=0$ and $adj\,A.B=0$ and A is matrix of order 3×3. Then system of equation $AX=B$ has infinitely many solutions or no solution. Incorrect option

(C) Since $AB=O$ and A is non-singular.
$\therefore\ B=O$. Correct option

(D) $|AB|\neq 0 \Rightarrow |A|\neq 0$ and $|B|\neq 0$
$\therefore\ A$ and B both are non-singular. Correct option

8. $x^6-2\lambda+1=x^6-2(x^5-x^3+x)+1$
$=(x-1)^1(x^4-x^2+1)\geq 0\ \ \forall\ x$
$\therefore$ option (A) and (B) are correct

9. Locus of the centre of the circle touching a line and a circle is always a parabola. Now as $y^2=8x$ is the locus of the centre of a circle touching a given circle and a given line. So, the centre of the given circle must be the focus of $y^2=8x$ i.e. $(2,0)$. Hence (A) option is correct and in particular case when given circle has radius 2 units then the given line must be y-axis which is tangent to the given circle hence (C) option is also correct.

10. $x-\{x\}=[x]\Rightarrow\{x-\{x\}\}=0\Rightarrow\{x-\{x-\{x\}\}\}=\{x\}$

$\Rightarrow\left\{[x-5]+\left\{x-\left\{x-\{x\}\right\}\right\}\right\}=\left\{5+[x]+\{x\}\right\}=\{5+x\}=\{x\}$

$=\begin{cases} x, & 0<x<1 \\ x-1, & 1\leq x\leq \pi/2\end{cases}$

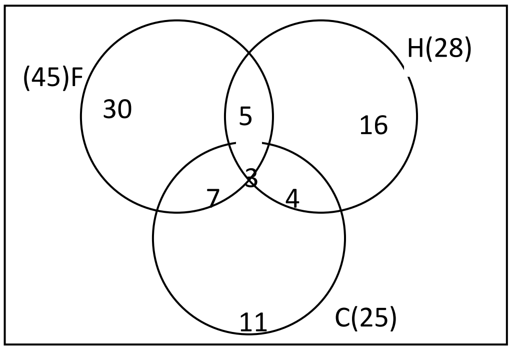
$\therefore\ f(x)=\begin{cases}\sin x, & 0<x<1\\ \sin(x-1), & 1\leq x\leq\pi/2\end{cases}$

$\therefore$ range $=(0,\sin1)\cup[0,\cos1]=[0,\sin1]$
$=(0,\sin1)\cup(0,\cos1)$

11. Let the angles be $a-d,a,a+d$
$\because\ (a-d)+(a)+(a+d)=180°$
$\Rightarrow\ 3a=180°\quad\Rightarrow\ a=60°$

$\therefore$ angles are $60°-d, 60°, 60°+d$
Also $2(60°-d)=60°+d$
$\Rightarrow\ 120°-2d=60°+d\ \Rightarrow\ 60°=3d,\ d=20°$
Required angles are 40°, 60°, 80°

12 TO 14



12. Total number of children $=45+16+4+11=76$

13. Number of ways of selecting a cricket team of 11 players $={}^{25}C_{11}$

14. Probability $=\dfrac{{}^4C_2}{{}^7C_2}=\dfrac{2}{7}$

15. (A) 3 . 1 – 2(-2) + 5(λ) = 0 $\Rightarrow\lambda=-\dfrac{7}{5}.$

(B) Point $(3,\lambda,\mu)$ lies on 2x + y + z – 3 = x – 2y + z – 1
$\Rightarrow 3.\ 2+\lambda+\mu-3=0$ and $3-2\lambda+\mu-1=0$
$\Rightarrow\lambda+\mu+3=0$ and $2\lambda-\mu-2=0$
so $\lambda+\mu=-3.$

(C) $\sin\theta=\dfrac{1.4+1(-3)+1\ .\ 5}{\sqrt{1^2+1^2+1^2}\sqrt{16+9+25}}=\dfrac{6}{\sqrt3\sqrt{50}}$

$\theta=\sin^{-1}\sqrt{\dfrac{6}{25}}.$

(D) $\cos\theta=\dfrac{1.3+1(-4)+1.5}{\sqrt3\sqrt{16+9+25}}=\dfrac{4}{\sqrt3\sqrt{50}}$

$\theta=\cos^{-1}\sqrt{\dfrac{8}{75}}.$

16. $\dfrac{dy}{dx}=x^2-2x\quad\Rightarrow\ y=\dfrac{x^3}{3}-x^2+C$
Passing through (2, 0)
$\Rightarrow 0=\dfrac{8}{3}-4+C\Rightarrow C=\dfrac{4}{3}\quad\therefore\ y=\dfrac{x^3}{3}-x^2+\dfrac{4}{3}$
$y'=x^2-2x=x(x-2)\Rightarrow y'=0$ if $x=2$
and at $x=2$, y takes the minimum value.

$\therefore$ minimum value of y is $=\frac{8}{3}-4+\frac{4}{3}=0$

$\therefore a=2,\ b=0 \qquad \therefore a+6b=2$

17. $f(x)=ae^{2x}+be^{x}+cx$

Since $f(0)=a+b$ i.e. $a+b=-1$ ...... (i)

$f'(x)=2ae^{2x}+be^{x}+c$

$\therefore \quad f'(\log 2)=2ae^{2\log 2}+be^{\log 2}+c$

$=8a+2b+c$

$=8a+2b+c=31$ ......(ii)

$$\int_0^{\ell n4}(ae^{2x}+b.e^{x}+cx-cx)dx=\int_0^{\ell n4}(ae^{2x}+be^{x})dx$$

$$=\left(\frac{ae^{2x}}{2}+be^{x}\right)_0^{\log 4}=\frac{ae^{2\ell n4}}{2}+be^{\ell n4}-\frac{a}{2}-b$$

$$=8a+4b-\frac{a}{2}-b=\frac{15a}{2}+3b=\frac{39}{2}$$

i.e. $15a+6b=39$ ......(iii)

from (i), (ii) and (iii) $9a=45$

$\therefore \ a=5,\quad b=-6$ and $c=3$

18. Let $BC=a=3$, $AC=b=4$ and $AB=c$

$(AG)^2+(BG)^2=AB^2, AG=\frac{2}{3}AA_1, BG=\frac{2}{3}BB_1$

$\frac{1}{9}[2b^2+2c^2-a^2+2a^2+2c^2-b^2]=c^2$

$\Rightarrow 5c^2=a^2+b^2=25$ Now,

$BG^2=BM.BA\Rightarrow \frac{1}{9}.(2a^2+2c^2-b^2)=BM.c$

$\Rightarrow BM=\frac{4}{3\sqrt{5}} \quad \therefore\ a.c.BM=3.\sqrt{5}.\frac{4}{3\sqrt{5}}=4$

19. $\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^{x}=$

$$e^{\lim_{x\to\infty}x\left(\sin\frac{1}{x}+\cos\frac{1}{x}-1\right)}=e^{\lim_{x\to\infty}\left(\frac{\sin(1/x)}{1/x}-x\left(2\sin^2\frac{1}{2x}\right)\right)}$$

$=e \quad \therefore\ k=2$

20. S_1: Let $z_1=\alpha_1+i\beta_1$ and $z_2=\alpha_2+i\beta_2$ where $\alpha_1,\beta_1,\alpha_2,\beta_2\in R$ and β_1 and β_2 are non zero.

$z_1+z_2=\alpha_1+i\beta_1+\alpha_2+i\beta_2=(\alpha_1+\alpha_2)+i(\beta_1+\beta_2)$

$z_1z_2=(\alpha_1+i\beta_1)(\alpha_2+i\beta_2)=\alpha_1\alpha_2-i\beta_1\beta_2+i(\alpha_1\beta_2+\alpha_2\beta_1)$

If z_1+z_2 and z_1z_2 are real, then $\beta_1+\beta_2=0$ and $\alpha_1\beta_2+\alpha_2\beta_1=0$

$\beta_1=-\beta_2\Rightarrow \alpha_1=\alpha_2 \qquad \therefore\ \alpha_1-i\beta_1$

S_2: Since $|z_1|=|z_2|=|z_3|=1$ $\therefore$ the points lie on unit circle with centre at origin. Since the triangle is the same as the circumcentre $\therefore$ centroid of the triangle is the same as the circumcentre $\therefore \frac{z_1+z_2+z_3}{3}=0 \quad \therefore S_2$ is true

S_3: if z_1,z_2,z_3 are in A.P., then $2z_2=z_1+z_3\Rightarrow z_2$ is the midpoint of the segment joining z_1 and z_2. Therefore z_1,z_2,z_3 lie on a straight line.

21. $S_1:[\vec a\,\vec b\,\vec c]=\lambda$

$[\vec a+\vec b\ \vec b+\vec c\ \vec c+\vec a]=(\vec a+\vec b).(\vec b\times\vec c+\vec b\times\vec a+\vec c\times\vec a)$

$[\vec a\,\vec b\,\vec c]+[\vec b\,\vec c\,\vec a]=2\lambda \quad \therefore S_1$ is true

$S_2: a,b,c$ are in A.P. $\Rightarrow \frac{a}{abc},\frac{b}{abc},\frac{c}{abc}$ are in A.P.

$\Rightarrow \frac{1}{bc},\frac{1}{ac},\frac{1}{ab}$ are in A.P.

$\Rightarrow a+\frac{1}{bc},b+\frac{1}{ac},c+\frac{1}{ab}$ are in A.P. $\therefore S_2$ is false

S_3: $f(x)+f\left(x+\frac{1}{2}\right)=1$

$$\int_0^1 f(x)\,dx=\int_0^{1/2}f(x)dx+\int_0^{1/2}f(1-x)dx$$

$$=\int_0^{1/2}f(x)dx+\int_0^{1/2}f\left(1-\left(\frac{1}{2}-x\right)\right)dx$$

$$=\int_0^{1/2}f(x)dx+\int_0^{1/2}f\left(\frac{1}{2}+x\right)dx$$

$$=\int_0^{1/2}f(x)dx+\int_0^{1/2}\left(1-f(x)\right)dx=\frac{1}{2}$$ $\therefore\ S_3$ is true

22. S_1: $\sqrt{4\sin^4\theta+\sin^2 2\theta}+4\cos^2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)$

$=\sqrt{\left(2\sin^2\theta\right)^2+\sin^2 2\theta}+2\left(1+\cos\left(\frac{\pi}{2}-\theta\right)\right)$

$=\sqrt{\left(1-\cos 2\theta\right)^2+\sin^2 2\theta}+2+2\sin\theta$

$=\sqrt{2-2\cos 2\theta}+2+2\sin\theta$

$=-2\sin\theta+2+2\sin\theta=2$ which is true.

S_2: false (coefficient may not be real)

$S_3:\ \frac{{}^nC_r}{1}=\frac{{}^nC_{r+1}}{7}=\frac{{}^nC_{r+2}}{42}$ i.e. $\frac{r+1}{n-r}=\frac{1}{7}$

and $\frac{r+2}{n-r-1}=\frac{1}{6}\ \therefore\ n-r=7r+7$ and

$n-r-1=6r+12\ \therefore\ 1=r-5\ \therefore\ r=6\ \therefore\ n=55$