1. (b) $n((A\times B)\cap(B\times A))$

$=n((A\cap B)\times(B\cap A))=n(A\cap B).n(B\cap A)$

$=n(A\cap B).n(A\cap B)=(99)(99)=99^2$.

2. (d) $\sqrt{50}+\sqrt{48}=5\sqrt{2}+4\sqrt{3}=\sqrt{2}[5+2.\sqrt{2}.\sqrt{3}]$

$=\sqrt{2}(\sqrt{3}+\sqrt{2})^2$; $\therefore\ \sqrt{\sqrt{50}+\sqrt{48}}=2^{1/4}(\sqrt{3}+\sqrt{2})$.

3. (a) $\frac{1+iz}{1-iz}=\frac{1+i(b+ic)/(1+a)}{1-i(b+ic)/(1+a)}=\frac{1+a-c+ib}{1+a+c-ib}$

$=\frac{(1+a-c+ib)(1+a+c+ib)}{(1+a+c)^2+b^2}$

$=\frac{1+2a+a^2-b^2-c^2+2ib+2iab)}{1+a^2+c^2+b^2+2ac+2(a+c)}$

$=\frac{a^2+b^2+c^2+2a+a^2-b^2-c^2+2ib(1+a)}{1+1+2ac+2(a+c)}$

$=\frac{2a(a+1)+2ib(1+a)}{2(1+a)(1+c)}=\frac{a+ib}{1+c}$.

4. (c) Let a be the first term and x be the common difference of the A.P. Then $a+5x=2\Rightarrow a=2-5x$

Let $P=a_1a_4a_5=a(a+3x)(a+4x)$

$=(2-5x)(2-2x)(2-x)=2(-5x^3+17x^2-16x+4)$

Now $\frac{dP}{dx}=0\Rightarrow x=\frac{8}{5},\frac{2}{3}$. Clearly, $\frac{d^2P}{dx^2}>0$ for $x=\frac{2}{3}$. Hence P is least for $x=\frac{2}{3}$.

5. (b) Obviously, roots will be equal in magnitude but opposite in sign if coefficient of $x=0$.

But the equation is $x^2+2mx+m^2-ab=0$

6. (d) Without any restriction the 10 persons can be ranked among themselves in $10!$ ways; but the number of ways in which A_1 is above A_{10} and the number of ways in which A_{10} is above A_1 make up $10!$. Also the number of ways in which A_1 is above A_{10} is exactly same as the number of ways in which A_{10} is above A_1.

Therefore the required number of ways $=\frac{1}{2}(10!)$.

7. (b) We have $(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+....+(x+2)^{n-1}$

$=\frac{(x+3)^n-(x+2)^n}{(x+3)-(x+2)}=(x+3)^n-(x+2)^n$

$(\because \frac{x^n-a^n}{x-a}=x^{n-1}+x^{n-2}a^1+x^{n-3}a^2+....+a^{n-1})$

Therefore coefficient of x^r in the given expression

= Coefficient of x^r in $[(x+3)^n-(x+2)^n]$

$={}^nC_r3^{n-r}-{}^nC_r2^{n-r}={}^nC_r(3^{n-r}-2^{n-r})$

8. (b) $\frac{1.2}{1!}+\frac{2.3}{2!}+\frac{3.4}{3!}+\frac{4.5}{4!}+.........\infty$,

Here $T_n=\frac{n(n+1)}{n!}=\frac{n+1}{(n-1)!}=\frac{(n-1)+2}{(n-1)!}$

$=\frac{1}{(n-2)!}+\frac{2}{(n-1)!}$

$\Rightarrow S=\Sigma T_n=e+2e=3e$.

9. (a) For unique solution of the given system $D\neq 0$

$$\begin{vmatrix}1 & 1 & 1\\ 5 & -1 & \mu\\ 2 & 3 & -1\end{vmatrix}\neq 0.$$

So this depends on μ only.

10. (d) We have

$e^{\log_{10}\tan 1^o+\log_{10}\tan 2^o+\log_{10}\tan 3^o+........+\log_{10}\tan 89^o}$

$=e^{\log_{10}(\tan 1^o\tan 2^o\ \tan 3^o.....\tan 89^o)}=e^{\log_{10}1}=e^o=1$

11. (b)

$c\cos(A-\alpha)+a\cos(C+\alpha)$

$=c(\cos A\cos\alpha+\sin A\sin\alpha)$

$+a(\cos C\cos\alpha-\sin C\sin\alpha)$

$=\cos\alpha(c\cos A+a\cos C)+c\sin A\sin\alpha-a\sin C\sin\alpha$

$\Rightarrow b\cos\alpha+kac\sin\alpha-kac\sin\alpha=b\cos\alpha$.

12. (b) $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{\pi}{2}$

Put $\sin^{-1}x=\alpha$, $\sin^{-1}y=\beta$, $\sin^{-1}z=\gamma$

$\therefore\ \alpha+\beta+\gamma=\frac{\pi}{2}$, (Given)

or $\alpha+\beta=\frac{\pi}{2}-\gamma$ or $\cos(\alpha+\beta)=\cos\left(\frac{\pi}{2}-\gamma\right)$

$\cos\alpha\cos\beta-\sin\alpha\sin\beta=\sin\gamma$ .....(i)

and, we have $\sin\alpha=x\Rightarrow\cos\alpha=\sqrt{1-x^2}$

Similarly, $\cos\beta=\sqrt{1-y^2}$

$\therefore$ From equation (i), we get $\sqrt{1-x^2}.\sqrt{1-y^2}=xy+z$

Squaring both sides, we have $x^2+y^2+z^2+2xyz=1$.

13. (b) Let $h=u\cos\alpha.t$, $k=u\sin\alpha.t-\frac{1}{2}gt^2$, then

$t=\frac{h}{u\cos\alpha}$. Putting the value of t in

$k=u\sin\alpha.t-\frac{1}{2}gt^2$, we get

$k=h\tan\alpha-\frac{1}{2}g\frac{h^2}{u^2\cos^2\alpha}$

$\therefore$ Locus of (h,k) is $y=x\tan\alpha-\frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$,

which is a parabola.

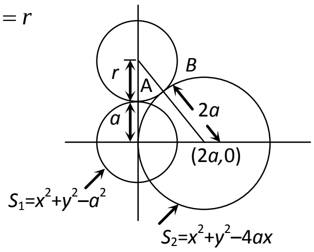
14. (a) Since the origin and the point (1, –3) lie on the same side of $x+2y-11=0$ and on the opposite side of $3x-6y-5=0$. Therefore, the bisector of the angle containing (1,–3) is the bisector of that angle which does not contain the origin and is given by $\frac{-x-2y+11}{\sqrt{5}}=-\left(\frac{-3x+6y+5}{\sqrt{45}}\right)$ *i.e.*, $3x=19$.

15. (a) If the equation of line is $y=mx$ and the length of perpendicular drawn on it from the point (x_1,y_1) is *d*, then

$\frac{y_1 - mx_1}{\sqrt{1+m^2}} = \pm d \Rightarrow (y_1 - mx_1)^2 = d^2(1+m^2)$. But

$m = \frac{y}{x}$, therefore on eliminating 'm' , the required equation is $(xy_1 - yx_1)^2 = d^2(x^2+y^2)$.

16. (a) Let $C \equiv (h, k)$, radius $= r$



Co-ordinates of $A \equiv \left[\frac{ah}{a+r}, \frac{ak}{a+r}\right]$

Co-ordinates of $B \equiv \left[\frac{2ar+2ah}{2a+r}, \frac{2ak}{2a+r}\right]$

Putting co-ordinates of A and B in S_1, S_2 respectively and eliminating r, we get the locus $12x^2 - 4y^2 - 24ax + 9a^2 = 0$.

Or : Since it touches $x^2 + y^2 = a^2$ and $x^2 + y^2 - 4ax = 0$, therefore

$r + a = \sqrt{h^2 + k^2}$ ....(i)

$r + 2a = \sqrt{(h-2a)^2 + k^2}$ ....(ii)

From (i), putting the value of r in (ii), we get

$-a + \sqrt{h^2+k^2} + 2a = \sqrt{(h-2a)^2 + k^2}$

On simplification, we get the required locus.

17. (a) Tangent at $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

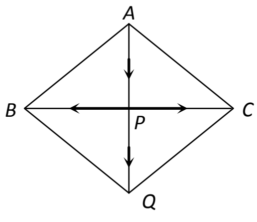
Therefore, tangent at $y^2 = 4a(x+a)$ is,

$y = m(x+a) + \frac{a}{m}$

or $y = mx + ma + \frac{a}{m} \Rightarrow ma + \frac{a}{m} = c$.

18. (c) $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PQ}$ or $\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PQ} + \overrightarrow{CP}$

or $\overrightarrow{AB} = \overrightarrow{CQ}$.



Hence it is a parallelogram.

19. (a) Here, three given lines are coplanar if they have common perpendicular

Let d.c.'s of common perpendicular be l, m, n

$\Rightarrow ll_1 + mm_1 + nn_1 = 0$ .....(i)

$ll_2 + mm_2 + nn_2 = 0$ .....(ii)

and $ll_3 + mm_3 + nn_3 = 0$ .....(iii)

Solving (ii) and (iii), we get

$\frac{l}{m_2n_3 - n_2m_3} = \frac{m}{n_2l_3 - n_3l_2} = \frac{n}{l_2m_3 - l_3m_2} = k$

$\Rightarrow$
$l = k(m_2n_3 - n_2m_3), m = k(n_2l_3 - n_3l_2), n = k(l_2m_3 - l_3m_2)$

Substituting in (i), we get

$l_1(m_2n_3 - n_2m_3) + m_1(n_2l_3 - n_3l_2) + n_1(l_2m_3 - l_3m_2) = 0$

$\Rightarrow \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = 0 \Rightarrow -\begin{vmatrix} l_1 & n_1 & m_1 \\ l_2 & n_2 & m_2 \\ l_3 & n_3 & m_3 \end{vmatrix} = 0$.

20. (a) Putting $x = \frac{1}{t}$, the given limit

$= \lim_{t \to 0} \frac{\frac{\sin t}{t} - 1}{t - 1} = \frac{1-1}{0-1} = 0$, which is given in (a).

Or : $\lim_{x \to \infty} \frac{x^2 \sin\frac{1}{x} - x}{1 - |x|}$

$= \lim_{x \to \infty} \frac{x^2\left(\frac{1}{x} - \frac{1}{3!}\frac{1}{x^3} + ...\right) - x}{1 - |x|}$, $\left[\because \frac{1}{x} \to 0\right]$

$= \lim_{x \to \infty} \frac{\left(x - \frac{1}{6x} + ... - x\right)}{1 - |x|}$

$= \lim_{x \to \infty} \frac{\frac{1}{6x} - \text{terms containing powers of } \frac{1}{x}}{|x| - 1} = 0.$

21. (b) Since $g(x)$ is the inverse of function $f(x)$, therefore $gof(x) = I(x)$ for all x.

Now $gof(x) = I(x), \ \forall x$

$\Rightarrow gof(x) = x, \ \forall x \Rightarrow (gof)'(x) = 1, \ \forall x$

$\Rightarrow g'(f(x))f'(x) = 1, \ \forall x$ (using chain rule)

$\Rightarrow g'(f(x)) = \frac{1}{f'(x)}, \ \forall x \Rightarrow g'(f(c)) = \frac{1}{f'(c)}$ (putting x=c)

22. (a) $\int (x+3)(x^2+6x+10)^9 dx$

$= \frac{1}{2}\int (2x+6)(x^2+6x+10)^9 dx$

$= \frac{1}{2}\frac{(x^2+6x+10)^{10}}{10} + c = \frac{1}{20}(x^2+6x+10)^{10} + c$.

23. (b) We have $F'(x) = 3\sin x + 4\cos x$

Since in $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right], F'(x) < 0,$ so assume the least value at the point $x = \frac{4\pi}{3}$.

Thus, $f\left(\frac{4\pi}{3}\right) = \int_{5\pi/4}^{4\pi/3} (3\sin u + 4\cos u) du$

$= \frac{3}{2} - 2\sqrt{3} + \frac{1}{\sqrt{2}}$.

24. (a) $\frac{d^2y}{dx^2} = \sec^2 x + xe^x$

On integrating, $\frac{dy}{dx} = \tan x + xe^x - e^x + c_1$

Again, $y = \log(\sec x) + xe^x - e^x - e^x + c_1 x + c_2$

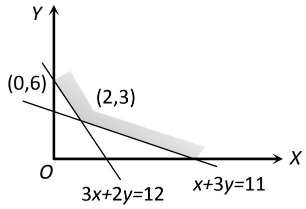
Thus required solution is

$y = \log(\sec x) + (x-2)e^x + c_1 x + c_2$.

25. (c) $P(\overline{A} \cap B) = P(B) - P(A \cap B) = y - z$.

26. (a) Let the mean of the remaining 4 observations be $\overline{x}_1$. Then, $M = \frac{a + 4\overline{x}_1}{(n-4)+4} \Rightarrow \overline{x_1} = \frac{nM - a}{4}$.

27. (a) *Min* $z = 2(2) + 2(3) \Rightarrow c = 10$.



28. (d) For the given circuit, Boolean polynomial is $(\sim p \wedge q) \vee (p \wedge \sim q)$.

29. (a) Let *e* be the identity element for the binary operation *o* defined on *z* given by $a \, o \, b = a + b - ab$

Then $a \, o \, e = a = e \, o \, a$ for all $a \in z$

$\Rightarrow a + e - ae = a$ for all $a \in z \Rightarrow e(1-a) = 0$ for all $a \in z \Rightarrow e = 0$.

So, 0 is the identity element for the binary operation *o* on *z*.

Let x be the inverse of $a \in z$. Then, $a \, o \, x = x \, o \, a = 0$

$\Rightarrow a + x - ax = 0 \Rightarrow x(1-a) = -a$

$\Rightarrow x = \frac{a}{a-1} \quad (\because a \neq 1)$

Thus, $\frac{a}{a-1}$ is the inverse of $a(\neq 1) \in z$.

30. (c)

$arg\left(\frac{z_1}{\overline{z}_2}\right) = arg z_1 - arg(\overline{z}_2) = arg z_1 + arg z_2$

$= arg(z_1 . z_2)$

Option (c) gives the same result.

31. (d) $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + \ldots\ldots + \log_{\sqrt[16]{3}} x = 36$

$\Rightarrow \frac{1}{\log_x \sqrt{3}} + \frac{1}{\log_x \sqrt[4]{3}} + \frac{1}{\log_x \sqrt[6]{3}} + \ldots + \frac{1}{\log_x \sqrt[16]{3}} = 36$

$\Rightarrow \frac{1}{(1/2)\log_x 3} + \frac{1}{(1/4)\log_x 3} + \frac{1}{(1/6)\log_x 3} + \ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots + \frac{1}{(1/16)\log_x 3} = 36$

$\Rightarrow (\log_3 x)(2 + 4 + 6 + \ldots\ldots + 16) = 36$

$\Rightarrow (\log_3 x)\frac{8}{2}[2 + 16] = 36 \Rightarrow \log_3 x = \frac{1}{2}$

$\Rightarrow x = 3^{1/2} \Rightarrow x = \sqrt{3}$.

32. (b) Sum of roots $\alpha + \beta = -(a+b)$ and $\alpha\beta = \frac{a^2 + b^2}{2}$

$\Rightarrow (\alpha + \beta)^2 = (a+b)^2$ and $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 2ab - (a^2 + b^2) = -(a-b)^2$

Now the required equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$x^2 - \{(\alpha+\beta)^2 + (\alpha-\beta)^2\} x + (\alpha+\beta)^2(\alpha-\beta)^2 = 0$

$\Rightarrow x^2 - \{(a+b)^2 - (a-b)^2\} x - (a+b)^2(a-b)^2 = 0$

$\Rightarrow x^2 - 4abx - (a^2 - b^2)^2 = 0$

33. (b) $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$, $A_{11} = 1, A_{21} = -2, A_{31} = 4$

$A_{12} = 4, A_{22} = 1, A_{32} = -2$, $A_{13} = -2, A_{23} = 4, A_{33} = 1$

$Adj(A) = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & -2 \\ -2 & 4 & 1 \end{bmatrix}$.

34. (a) $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B$

$\Rightarrow \frac{2\cos\frac{A+C}{2}\sin\frac{A-C}{2}}{2\sin\frac{A+C}{2}\sin\frac{A-C}{2}} = \cot B$

$\Rightarrow \cot\frac{(A+C)}{2} = \cot B \Rightarrow B = \frac{A+C}{2}$

Thus *A, B, C* will be in A.P.

35. (c) The first equation can be written as

$2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y) = 2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x+y)$

$\therefore$ Either $\sin\frac{1}{2}(x+y) = 0$ or $\sin\frac{1}{2}x = 0$ or $\sin\frac{1}{2}y = 0$. Thus $x + y = -1,\ x - y = -1$.

When $x + y = 0$, we have to reject $x + y = 1$ and check with the options or $x + y = -1$ and solve it with $x - y = 1$ or $x - y = -1$ which gives $\left(\frac{1}{2}, -\frac{1}{2}\right)$ or $\left(-\frac{1}{2}, \frac{1}{2}\right)$ as the possible solution. Again solving with $x = 0$, we get $(0, \pm 1)$ and solving with $y = 0$, we get $(\pm 1, 0)$ as the other solution. Thus we have six pairs of solution for x and y.

36. (d) Since $S(3, 2) = 9 + 4 - 25 < 0$, therefore (3, 2) lies inside the circle. So there exists no chord of contact and hence ΔOAB does no exist.

37. (b) $\frac{2b^2}{a} = 2ae \Rightarrow b^2 = a^2 e$ or $e = \frac{b^2}{a^2}$

Also $e = \sqrt{1 - \frac{b^2}{a^2}}$ or $e^2 = 1 - e$ or $e^2 + e - 1 = 0$

Therefore $e = \frac{-1 \pm \sqrt{5}}{2}$. As $e < 1$, $\therefore e = \frac{\sqrt{5} - 1}{2}$.

38. (c) $\int_0^1 e^{x^2}(x - \alpha)dx = 0 \Rightarrow \frac{1}{2}\int_0^1 2x.e^{x^2}dx = \alpha\int_0^1 e^{x^2}dx$

$\Rightarrow \frac{1}{2}| e^{x^2} |_0^1 = \alpha\int_0^1 e^{x^2}dx \Rightarrow \frac{1}{2}(e-1) = \alpha\int_0^1 e^{x^2}dx$

$\Rightarrow \alpha = \dfrac{\frac{1}{2}(e-1)}{\int_0^1 e^{x^2}dx} > 0$ and $\alpha < 1$. So, $0 < \alpha < 1$.

39. (a) Here $(l\mathbf{a} + m\mathbf{b}) \times \mathbf{b} = \mathbf{c} \times \mathbf{b} \Rightarrow l\mathbf{a} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$

$\Rightarrow l(\mathbf{a} \times \mathbf{b})^2 = (\mathbf{c} \times \mathbf{b}).(\mathbf{a} \times \mathbf{b}) \Rightarrow l = \dfrac{(\mathbf{c} \times \mathbf{b}).(\mathbf{a} \times \mathbf{b})}{(\mathbf{a} \times \mathbf{b})^2}$

Similarly, $m = \dfrac{(\mathbf{c} \times \mathbf{a}).(\mathbf{b} \times \mathbf{a})}{(\mathbf{b} \times \mathbf{a})^2}$.

40. (b) We have $f(x) = \begin{cases} \dfrac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} \dfrac{x^2}{x} = x, & x > 0 \\ 0, & x = 0 \\ \dfrac{x^2}{-x} = -x, & x < 0 \end{cases}$

We have
$\lim_{x \to 0-} f(x) = \lim_{x \to 0} -x = 0$, $\lim_{x \to 0+} f(x) = \lim_{x \to 0} x = 0$ and $f(0) = 0$.
So $f(x)$ is continuous at $x = 0$.
Also $f(x)$ is continuous for all other values of *x*.
Hence, $f(x)$ is continuous everywhere. Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$. Therefore $f(x)$ is not differentiable at $x = 0$.

41. (d) $y = 2\cos 2x - \cos 4x$

$= 2\cos 2x(1 - \cos 2x) + 1 = 4\cos 2x \sin^2 x + 1$

Obviously, $\sin^2 x \geq 0$
Therefore to be least value of *y*, $\cos 2x$ should be least *i.e.,* -1. Hence least value of *y* is $-4 + 1 = -3$.

42. (c) As $f(x) = \sin 2x \Rightarrow f'(x) = 2\cos 2x$

Obviously $f'(x) > 0$ in $\left(0, \dfrac{\pi}{4}\right)$ and $f'(x) < 0$ in $\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$

43. (a) Putting $\tan^{-1} x = t$ and $\dfrac{dx}{1+x^2} = dt$, we get

$\int e^{\tan^{-1} x}\left(\dfrac{1+x+x^2}{1+x^2}\right)dx = \int e^t(\tan t + \sec^2 t)dt$

$= e^t \tan t + c = x\, e^{\tan^{-1} x} + c$

$\left[\text{Using} \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$.

44. (b) $2a = 10$, $\therefore a = 5$

$ae - a = 8$ or $e = 1 + \dfrac{8}{5} = \dfrac{13}{5}$

$\therefore b = 5\sqrt{\dfrac{13^2}{5^2} - 1} = 5 \times \dfrac{12}{5} = 12$ and centre of hyperbola $\equiv (5, 0)$ $\therefore \dfrac{(x-5)^2}{5^2} - \dfrac{(y-0)^2}{12^2} = 1$.

45. (a) The total number of ways of selecting 4 tickets $= 3^4 = 81$.
The favourable number of ways
= sum of coefficients of $x^2, x^4, \ldots\ldots$ in $(x + x^2 + x^3)^4$
= sum of coefficients of $x^2, x^4, \ldots\ldots$ in $x^4(1 + x + x^2)^4$.

Let $(1 + x + x^2)^4 = 1 + a_1 x + a_2 x^2 + \ldots.. + a_8 x^8$.

Then $3^4 = 1 + a_1 + a_2 + a_3 + \ldots. + a_8$, (On putting $x = 1$)
and $1 = 1 - a_1 + a_2 - a_3 + \ldots.. + a_8$, (On putting $x = -1$)
$\therefore 3^4 + 1 = 2(1 + a_2 + a_4 + a_6 + a_8)$
$\Rightarrow a_2 + a_4 + a_6 + a_8 = 41$
Thus sum of the coefficients of $x^2, x^4, \ldots\ldots = 41$
Hence the required probaility $= \dfrac{41}{81}$.