

# Feel Mathematics

## FULL TEST -1 for CBSE XII Mathematics

Time : 3 hrs

MM : 100

Instructions : (a) Answer all questions.

(b) Internal choices are provided for some questions.

(c) Question numbers **1** to **10** are very short answer questions and carry **1 mark** each.

(d) Question numbers **11** to **22** are short answer questions and carry **4 marks** each.

(e) Question numbers **23** to **29** are long answer questions and carry **6 marks** each.

(f) Use of calculator is not permitted. However you may ask for logarithmic and statistical tables, if required.

(g) Use graph paper for Linear Programming Problem.

1. Without expanding the determinant, prove that 
$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0.$$

2. Find the projection of  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  on  $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ .

3. If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , show that  $A(\text{adj } A) = |A| I_2$

4. Find the number of all one-one functions from set  $A = \{1, 2, 3\}$  to itself.

5. Evaluate :  $\int \sqrt{x}(2x^3 + 5x + 7) dx$ .

6. Two lines have direction ratios 3, 2, -1 and -2, -1, -8 respectively. Find the angle between these lines.

7. Show that,  $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1} x$ .

8. If  $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$  and  $B = [-2 \ -1 \ -4]$ , verify that  $(AB)' = B'A'$

9. Evaluate :  $\int_0^{\pi/4} \tan x dx$ .

10. If the lines  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$  and  $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$  intersect, then find the value of k

11. If  $R_1$  and  $R_2$  are equivalence relations in a set A, show that  $R_1 \cap R_2$  is also an equivalence relation.

# Feel Mathematics

12. Evaluate:  $\int_1^4 f(x) dx$ , where  $f(x) = |x - 1| + |x - 2| + |x - 3|$

13. Solve the differential equation :  $x \frac{dy}{dx} - y (\log y - \log x + 1) = 0$ .

14. A bag contains 11 tickets numbered from 1 to 11. Two tickets are drawn without replacement. What is the probability that the second ticket has an even number given that the first has an odd number ?

OR

determine the binomial distribution for which the mean is 4 and standard deviation is 3.

15. Differentiate  $\log \left[ \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} \right]$  w.r.t. x.

OR

If  $\tan(x + y) + \tan(x - y) = 1$ , find  $dy/dx$ .

16. Using vectors, show that angle in a semi-circle is a right angle.

17. Find the foot of the perpendicular from the point  $(3, -1, 11)$  on the line  $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ .

Also find the length of the perpendicular.

18. Expression in the simplest form :  $\tan^{-1} \left( \frac{3a^2x - x^3}{a^3 - 3ax^2} \right)$ ,  $a > 0$ ,  $\frac{-a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$ .

OR

Show that  $\sin^{-1} \left( \frac{12}{13} \right) + \cos^{-1} \left( \frac{4}{5} \right) + \tan^{-1} \left( \frac{63}{16} \right) = \pi$ .

19. Find the equation of the tangent and normal to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  at the point  $(x_0, y_0)$ .

OR

The length  $x$  of a rectangle is decreasing at the rate of 5 cm/minute and the width  $y$  is increasing at the rate of 4 cm/minute. When  $x = 8$  cm and  $y = 6$  cm, find the rate of change of (a) the perimeter and (b) the area of the rectangle.

20. If  $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$  then find the value of  $f(100)$

21. Using integration, find the area of the triangular region whose vertices are P (1, 0), Q (2, 2) and R (3, 1).

OR

Evaluate :  $\int_1^4 (x^2 - x) dx$  as limit of a sum.

# Feel Mathematics

22. Let  $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$ .

Determine the value of  $a$  if possible, so that the function is continuous at  $x = 0$

23. Show that the height of a closed circular cylinder of given total surface area and maximum volume is equal to the diameter of the base.

24. Evaluate :  $\int \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}} dx$ .

25. For any two vectors  $\vec{u}$  and  $\vec{v}$  prove that :

(a)  $|\vec{u} \cdot \vec{v}|^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2$  and

(b)  $(1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = |1 - \vec{u} \cdot \vec{v}|^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$

26. If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ , find  $A^{-1}$  and use it to solve the system of equations

$$x + y + 2z = 0$$

$$x + 2y - z = 9$$

$$x - 3y + 3z = -14.$$

27. A home decorator manufactures two types of lamps A and B. Lamp A requires 2 hours of the cutter's time and 1 hour of finisher. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 104 hours and finisher has 76 hours of time available each month. Profit on A is Rs.6 and on B is Rs.11. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit? Solve it graphically.

28. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one day by one without replacement and are tested till all defective articles are found. What is the probability that the testing procedure ends at the twelfth testing?

29. Find the area bounded by the curves  $x^2 = y$ ,  $x^2 = -y$  and  $y^2 = 4x - 3$