1. (c)

We have for $\cos^{-1}(1-x) \geq 0$

$\Rightarrow -1 \leq (1-x) \leq 1 \Rightarrow \quad 0 \leq x \leq 2$ ....(1)

also $\quad 10\cdot3^{x-2} - 9^{x-1} - 1 > 0$

$10\cdot3^{x} - 9^{x} - 9 > 0 \quad \Rightarrow \quad 10\cdot3^{x} - 3^{2x} - 9 > 0$

$3^{2x} - 10\cdot3^{x} + 9 < 0 \quad \Rightarrow \quad (3^{x}-1)(3^{x}-9) < 0$

$1 < 3^{x} < 9 \quad \Rightarrow \quad 0 < x < 2$ .....(2)

from (1) and (2) $\quad 0 < x < 2$

2. (d)

$\tan\alpha = 3$; $\tan\beta = 2$.

Now $\sin 2(\alpha - \beta) = \sin 2\alpha \cos 2\beta - \cos 2\alpha \sin 2\beta$

$= \frac{2\,.\,3}{1+9}\cdot\frac{1-4}{1+4} - \frac{1-9}{1+9}\cdot\frac{2\,.\,2}{1+4} = \frac{7}{25}$

3. (d)

$T_{r+1} = {}^{18}C_r \cdot (9x)^{18-r} \cdot (-1)^r \cdot \frac{(x)^{-r/2}}{3^r}$

$T_{r+1} = {}^{18}C_r \cdot 9^{18-r} \cdot \frac{(-1)^2}{3^r} \cdot x^{18} \cdot \frac{3r}{2}$

$\Rightarrow 18 - \frac{3r}{2} = 0 \quad \Rightarrow r = 12$

${}^{18}C_2 \cdot \frac{9^6}{3^{12}} = k \cdot {}^{18}C_2 \quad \Rightarrow \quad k = 1$

4. (b)

5. (c)

Since $\cos x = \tan x = \frac{\sin x}{\cos x}$, the angle x is in the first or second quadrant. Now $\cos^2 x = \sin x$. Thus $\sin x$ is non-negative and substituting $\cos^2 x = 1 - \sin^2 x$ in the second equation result in $\sin^2 x + \sin x - 1 = 0$. The only non-negative solution of this equation is $\sin x = \frac{-1+\sqrt{5}}{2}$

6. (b)

$\text{Im}\,(z-2)(\bar{z}+i) = 0$; $\quad$ Real + i(2y + x − 2) = 0

7. (d)

8. (d)

$T_n = 1 - \frac{2}{n+2} = \frac{n}{n+2}$

$P = \prod_{n=1}^{98}\frac{n}{n+2} = \frac{1\cdot2\cdot3\cdot4..........\cdot97\cdot98}{3\cdot4\cdot5\cdot6..........\cdot98\cdot99\cdot100}$

$= \frac{1}{99\cdot100} = \frac{1}{4950}$

9. (c)

Let $x = 2000^{2000}$

$\log x = 2000 \cdot \log_{10}(2000) = 2000\,(\log_{10}2 + 3) = 2000\,(3.3010) = 6602$ ∴ number of digits = 6603

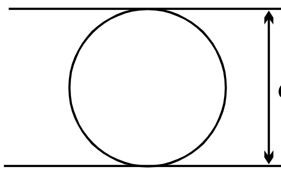
10. (d)

$\left.\begin{array}{l}\vec{a}\,.\,(\vec{b}+\vec{c}) = 0\\ \vec{b}\,.\,(\vec{c}+\vec{a}) = 0\\ \vec{c}\,.\,(\vec{a}+\vec{b}) = 0\end{array}\right\} \Rightarrow \vec{a}\,.\,\vec{b} + \vec{b}\,.\,\vec{c} + \vec{c}\,.\,\vec{a} = 0$

$\Rightarrow \vec{a}\,.\,\vec{b} = \vec{b}\,.\,\vec{c} = \vec{c}\,.\,\vec{a} = 0$

now square $|\vec{a}+\vec{b}+\vec{c}|$ to get the result

11. (d)



Line pair is

$2x - y + 3 = 0$

and $2x - y - 12 = 0$

$d = \left|\frac{15}{\sqrt{5}}\right| = 2r$

$\Rightarrow r = \frac{3\sqrt{5}}{2}$

12. (b)

$1 + \frac{a}{1-\frac{1}{1-a}} = 1 + \frac{a(1-a)}{1-a-1} = \frac{-a+a-a^2}{-a} = a$

$\Rightarrow a \neq 0, 1$

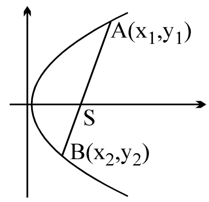
13. (a)

$u(x) = 7v(x) \quad \Rightarrow \quad u'(x) = 7v'(x)$

and $\frac{u(x)}{v(x)} = 7 \quad \Rightarrow \quad \left(\frac{u(x)}{v(x)}\right)' = 0$

$\Rightarrow \frac{p+q}{p-q} = \frac{7+0}{7-0} = 1$

14. (c)



$y^2 = 4ax$, $4a = 2p > 0$

$x_1 = at_1^2$, $y_1 = 2at_1$

$x_2 = at_2^2$, $y_2 = 2at_2$

and $\quad t_1t_2 = -1$

ratio $= \frac{4a^2t_1t_2}{a^2t_1^2t_2^2} = -4$

15. (a)

16. (d)

17. (b)

18. (a)

19. (a)

20. (d)

21. (d)

$y = mx \pm \sqrt{25m^2+16}$

$\sqrt{25m^2+16} = \dfrac{\pm\sqrt{25m^2+16}}{m}$ = k

$\Rightarrow 25m^2+16 = k^2$; $25m^2+16 = m^2k^2$

$k^2 = m^2k^2$; m^2 = 1 so k^2 = 25 + 16 = 41

22. (d)

$\dfrac{xdy - ydx}{x^2+y^2}$ + dx = 0 $\Rightarrow$ $\dfrac{x^2 d(y/x)}{x^2+y^2}$ + dx = 0

$\dfrac{d(y/x)}{1+(y/x)^2}$ + dx = 0 $\Rightarrow$ $\tan^{-1}(y/x)$ + x = C

23. (d)

AP = $\dfrac{30}{\cos 30°} = \dfrac{60}{\sqrt{3}}$ $\Rightarrow$ $\dfrac{h}{60/\sqrt{3}} = \dfrac{1}{\sqrt{3}}$ $\Rightarrow$ h = 20

24. (d)

| A | = 1 (1 + 2) – 2(– 1 – 4) – (1 –2)

= 3 + 10 + 1 = 14 $\Rightarrow$ (adjadjA) = $|A|^{(3-1)^2} = (14)^4$

25. (a)

$a_1 = 1, a_2 = r, a_3 = r^2, ...$ $\therefore 4a_2 + 5a_3 = 4r + 5r^2$

To be its minimum $\dfrac{d}{dr}(4r+5r^2) = 0 \Rightarrow r = \dfrac{-2}{5}$.

26. (a)

We have $\cos A = \dfrac{c^2+b^2-a^2}{2bc}$

$\Rightarrow c^2 - 2bc\cos A + (b^2 - a^2) = 0$

It is given that c_1 and c_2 are roots of this equation.

Therefore $c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$

$\Rightarrow$ $k(\sin C_1 + \sin C_2) = 2k\sin B\cos A$

$\Rightarrow$ $\sin C_1 + \sin C_2 = 2\sin B\cos A$

$\Rightarrow$ Now sum of the areas of two triangles

= $\dfrac{1}{2}ab\sin C_1 + \dfrac{1}{2}ab\sin C_2$ = $\dfrac{1}{2}ab(\sin C_1 + \sin C_2)$

$= \dfrac{1}{2}ab(2\sin B\cos A) = ab\sin B\cos A$

= $b(a\sin B)\cos A = b(b\sin A)\cos A = \dfrac{1}{2}b^2\sin 2A$.

27. (b)

$$3\cos\theta + 4\sin\theta = 5\left[\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right] = 5\cos(\theta - \alpha)$$

where $\cos\alpha = \dfrac{3}{5}$, $\sin\alpha = \dfrac{4}{5}$ Now

$3\cos\theta + 4\sin\theta = k$

$\therefore$ $5\cos(\theta-\alpha) = k \Rightarrow \cos(\theta - \alpha) = \pm 1$

$\Rightarrow$ $\theta - \alpha = 0^o, 180^o \Rightarrow \theta = \alpha,\ 180^o + \alpha$.

28. (c)

Since $-1 \le x < 0$. $\therefore [x] = -1$

Also $0 \le y < 1$. $\therefore [y] = 0$ And $1 \le z < 2$

$\therefore [z] = 1$. So $\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{vmatrix} = 1 = [z]$

29. (c)

Let $f(x) = \sin x - bx + c$

$\therefore f'(x) = \cos x - b > 0$ or $\cos x > b$ or $b < -1$.

30. (b)

Required area is $\int_0^a y\,dx = \int_0^a xe^{x^2}dx$

We put $x^2 = t \Rightarrow dx = \dfrac{dt}{2x}$ as $x = 0 \Rightarrow t = 0$ and $x = a \Rightarrow t = a^2$, then it reduces to

$\dfrac{1}{2}\int_0^{a^2} e^t dt = \dfrac{1}{2}[e^t]_0^{a^2} = \dfrac{e^{a^2}-1}{2}$ sq. unit.